

Form factor of $\pi^0 \rightarrow \gamma\gamma^*$

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Abstract

An effective chiral theory of large N_C QCD of mesons has been applied to study the form factor of $\pi\gamma\gamma^*$. Besides the poles of vector mesons an intrinsic form factor is found. The slope of the form factor is predicted. The effect of the current quark masses on the decay rate is calculated. There is no adjustable parameter in this study.

It is well known that the decay amplitude of $\pi^0 \rightarrow \gamma\gamma$ has been predicted by Adler-Bell-Jackiw[1], the famous triangle anomaly. This decay is a very important process in both particle physics and field theory. The form factor of $\pi\gamma\gamma^*$ is associated with both anomaly and Vector Meson Dominance(VMD). The measurements of the form factor has lasted for a long time[2,3]. In the timelike region of the slope of the form factor of $\pi^0 \rightarrow \gamma e^+ e^-$

$$F(q^2) = 1 + a \frac{m_{e^+e^-}^2}{m_{\pi^0}^2}$$

has been measured[3] and the value of the slope a is in a wide range

$$-0.24 \leq a \leq 0.12.$$

Recently, the PrimEx Collaboration of JLab proposes to do direct precision measurements of the slope a in a range of $0.001 GeV^2 \leq q^2 \leq 0.5 GeV^2$ [4]. On the other hand, the form factor of $\pi^0\gamma\gamma^*$ has been studied by various theoretical approaches[5].

In Ref.[6] we have proposed an effective chiral theory of large N_C QCD of pseudoscalar, vector, and axial-vector mesons. This theory is based on t'Hooft's large N_C expansion of QCD[7]. As interpolating fields, meson fields are coupled to quarks and they are not independent degrees of freedom. In the limit $m_q \rightarrow 0$, the effective theory is chiral symmetric and has dynamical chiral symmetry breaking. The Lagrangian is expressed as[6]

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a\gamma_5 + eQ\gamma \cdot A - mu(x))\psi(x) - \bar{\psi}(x)M\psi(x) \\ & + \frac{1}{2}m_0^2(\rho_i^\mu \rho_{\mu i} + \omega^\mu \omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu) \end{aligned} \quad (1)$$

where M is the quark mass matrix

$$\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix},$$

$v_\mu = \tau_i \rho_\mu^i + \omega_\mu$, $a_\mu = \tau_i a_\mu^i + f_\mu$, and $u = \exp\{i\gamma_5(\tau_i \pi_i + \eta)\}$. The kinetic terms of mesons are generated by quark loops. The Lagrangian of mesons is obtained by integrate the quark fields out. After renormalization physical meson fields, pion decay constant f_π , and a universal coupling constant g are defined. f_π and g are two inputs. g is determined to be 0.395 by fitting the decay rate of $\rho \rightarrow ee^+$. N_C expansion is revealed from this theory. The tree diagrams are at leading order of N_C expansion and loop diagrams of mesons are at higher orders. Adler-Bell-Jakiw(ABJ) and Wess-Zumino-Witten(WZW)[8] anomaly is the imaginary part of the Lagrangian of mesons[6]. VMD is a natural result of this theory[6]. We have applied this theory to study various meson physics and the theory is phenomenological successful[9,10].

The form factors of charged pion and kaons have been studied by this theory[9]. It has found that besides the poles of vector mesons, there are additional form factors which are called intrinsic form factors caused by quark loop effects. For example, the form factor of charged pion is determined to be[9]

$$F_\pi(q^2) = f_{\rho\pi\pi}(q^2) \frac{-m_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2)}{q^2 - m_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2)}, \quad (2)$$

where $\Gamma_\rho(q^2)$ is the decay width of ρ [9]. In the space-like region $\Gamma_\rho = 0$. Besides a ρ pole,

there is an intrinsic form factor $f_{\rho\pi\pi}(q^2)$

$$f_{\rho\pi\pi}(q^2) = 1 + \frac{q^2}{2\pi^2 f_\pi^2} \left\{ \left(1 - \frac{2c}{g}\right)^2 - 4\pi^2 c^2 \right\}, \quad (3)$$

$$c = \frac{f_\pi^2}{2gm_\rho^2}.$$

The effects of the intrinsic form factor are: a larger radius of charged pion is revealed from Eq.(2); in timelike region the $|F_\pi|$ decreases faster and in spacelike region F_π decreases slower than the ρ pole only. These results agree well with data.

In Ref.[6] we have studied the form factor of $\pi^0\gamma\gamma^*$. However, only the poles of vector mesons are taken into account. In this paper the intrinsic form factor of $\pi^0\gamma\gamma^*$ is studied and the slope is predicted. The effect of current quark mass on the decay rate of $\pi^0 \rightarrow \gamma\gamma$ is investigated too. In these studies there is no adjustable parameter.

There are four processes which contribute to the form factor of $\pi^0\gamma\gamma^*$. The four processes are shown in Fig.1. In these processes there are four vertices: $\mathcal{L}^{\pi^0\omega\rho}$, $\mathcal{L}^{\gamma\rho}$, $\mathcal{L}^{\gamma\omega}$, and $\mathcal{L}^{\pi^0\gamma\gamma}$. In Ref.[6] up to the fourth order in derivatives these vertices have been derived from Eq.(1). ABJ anomaly is obtained

$$\mathcal{L}_{\pi^0 \rightarrow \gamma\gamma} = -\frac{\alpha}{\pi f_\pi} \varepsilon^{\mu\nu\lambda\beta} \pi^0 \partial_\mu A_\nu \partial_\lambda A_\beta \quad (4)$$

and it is found that the form factor of $\pi^0 \rightarrow \gamma\gamma^*$ is the poles of ρ and ω mesons. However, like the form factor of charged pion(2), because of quark loop effects besides the poles of vector mesons we should expect an additional form factor for $\pi^0 \rightarrow \gamma\gamma^*$ too. In order to

investigate this intrinsic form factor we need to derive the four vertices to sixth order in derivatives. From Eq.(1) related interaction Lagrangian is obtained

$$\mathcal{L} = \bar{\psi} \left\{ \frac{1}{g} \gamma \cdot (\tau_3 \rho^0 + \omega) - i \frac{2m}{f_\pi} \tau_3 \gamma_5 \pi^0 - \frac{2}{f_\pi} \frac{c}{g} \tau_3 \gamma_\mu \gamma_5 \partial^\mu \pi^0 + eQ \gamma_\mu A^\mu \right\} \psi. \quad (5)$$

The term $-\frac{c}{g} \partial_\mu \pi^0$ is obtained from the transformation[6]

$$a_\mu^0 \rightarrow a_\mu^0 - \frac{2}{f_\pi} \frac{c}{g} \partial_\mu \pi^0$$

which is used to cancel the mixing between a_μ and π fields. In Ref.[6] the formalism of the path integral has been used to calculate the quark loop diagrams up to the fourth order in derivatives. In this paper we use the interaction Lagrangian(5) to calculate quark loop diagrams to find the meson vertices up to the sixth order in derivatives. We obtain

$$\mathcal{L}_{\rho\gamma} = -\frac{e}{4} g (\partial_\mu A_\nu - \partial_\nu A_\mu) \left\{ 1 - \frac{1}{10\pi^2 g^2} \frac{\partial^2}{m^2} \right\} (\partial^\mu \rho^{0\nu} - \partial_\nu \rho^{0\mu}), \quad (6)$$

$$\mathcal{L}_{\omega\gamma} = -\frac{e}{12} g (\partial_\mu A_\nu - \partial_\nu A_\mu) \left\{ 1 - \frac{1}{10\pi^2 g^2} \frac{\partial^2}{m^2} \right\} (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu), \quad (7)$$

$$\mathcal{L}_{\pi^0\omega\rho} = -\frac{3}{\pi^2 g^2 f_\pi} \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu \rho_\nu^0 \partial_\lambda \omega_\beta \left\{ 1 + \frac{1}{12m^2} \left(1 - \frac{2c}{g} \right) (k_1^2 + k_2^2 + p^2) \right\}, \quad (8)$$

where $k_{1,2}^2$ and p^2 are momentum(in fact, it is $-\partial^2$) of ρ , ω , and π^0 mesons respectively. It is interesting to notice that in Eq.(5) there is a coupling between $\partial_\mu \pi^0$ and the axial-vector current $\bar{\psi} \tau_3 \gamma_\mu \gamma_5 \psi$. The term $\frac{1}{12m^2} (-\frac{2c}{g}) (k_1^2 + k_2^2 + p^2)$ is obtained from this coupling. Because of the strong anomaly[10] this term is not zero in the chiral limit.

In deriving Eqs.(6-8) the equation[6] with a cut off

$$\frac{N_C}{(4\pi)^2} \frac{D}{4} m^2 \int d^D p \frac{1}{(p^2 + m^2)^2} = \frac{F^2}{16} \quad (9)$$

has been used. Following equations[6] are useful in this paper

$$F^2(1 - \frac{2c}{g}) = f_\pi^2, \quad (10)$$

$$m^2 = \frac{1}{6} \frac{F^2}{g^2}. \quad (11)$$

Using the substitutions

$$\rho_\mu^0 \rightarrow \frac{1}{2} eg A_\mu, \quad \omega_\mu \rightarrow \frac{1}{6} eg A_\mu \quad (12)$$

in Eq.(8), we obtain

$$\mathcal{L}_{\pi^0 \gamma \gamma} = -\frac{e^2}{4\pi^2 f_\pi} \left\{ 1 + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 (k_1^2 + k_2^2 + p^2) \right\} \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu A_\nu \partial_\lambda A_\beta, \quad (13)$$

$$\mathcal{L}_{\pi^0 \rho \gamma} = -\frac{e}{2g\pi^2 f_\pi} \left\{ 1 + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 (k_1^2 + k_2^2 + p^2) \right\} \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu A_\nu \partial_\lambda \rho_\beta, \quad (14)$$

$$\mathcal{L}_{\pi^0 \omega \gamma} = -\frac{3e}{2g\pi^2 f_\pi} \left\{ 1 + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 (k_1^2 + k_2^2 + p^2) \right\} \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu A_\nu \partial_\lambda \omega_\beta. \quad (15)$$

Using Eqs.(6-8,13-15), the amplitudes of the processes of Fig.1 are calculated in the chiral limit

$$\langle \gamma_1 \gamma_2 | S | \pi^0 \rangle = -i(2\pi)^4 \delta^4(p - k_1 - k_2) \frac{1}{\sqrt{8m_\pi \omega_1 \omega_2}} \varepsilon^{\mu\nu\lambda\beta} \epsilon_\mu(1) \epsilon_\nu(2) k_{1\lambda} k_{2\beta} \frac{2\alpha}{\pi f_\pi} F, \quad (16)$$

where

$$F = \left\{ 1 + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 (k_1^2 + k_2^2) \right\} \left\{ 1 - \frac{1}{2} \frac{k_1^2}{k_1^2 - m_\rho^2 + i\sqrt{k_1^2} \Gamma_\rho(k_1^2)} \right\}$$

$$\begin{aligned}
& -\frac{1}{2} \frac{k_2^2}{k_2^2 - m_\rho^2 + i\sqrt{k_2^2}\Gamma_\rho(k_2^2)} - \frac{1}{2} \frac{k_1^2}{k_1^2 - m_\omega^2 + i\sqrt{k_1^2}\Gamma_\omega(k_1^2)} \\
& -\frac{1}{2} \frac{k_2^2}{k_2^2 - m_\omega^2 + i\sqrt{k_2^2}\Gamma_\omega(k_2^2)} \},
\end{aligned} \tag{17}$$

where Γ_ρ and Γ_ω are the total decay width of ρ and ω meson respectively. The diagram Fig.1(d) is at higher order in derivatives. Therefore, the contribution of this diagram is not included in Eq.(17). In spacelike region they are zero. In timelike region[9]

$$\begin{aligned}
\Gamma_\rho(q^2) &= \Gamma_{\rho^0 \rightarrow \pi^+ \pi^-}(q^2) + \Gamma_{\rho^0 \rightarrow K \bar{K}}(q^2), \\
\Gamma_{\rho^0 \rightarrow \pi^+ \pi^-}(q^2) &= \frac{f_{\rho\pi\pi}^2(q^2)\sqrt{q^2}}{12\pi g^2} \left(1 - \frac{4m_{\pi^+}^2}{q^2}\right)^{\frac{3}{2}} \theta(q^2 > 4m_{\pi^+}^2), \\
\Gamma_{\rho^0 \rightarrow K \bar{K}}(q^2) &= \frac{f_{\rho\pi\pi}^2(q^2)\sqrt{q^2}}{48\pi g^2} \left(1 - \frac{4m_{K^+}^2}{q^2}\right)^{\frac{3}{2}} \theta(q^2 > 4m_{K^+}^2) \\
&\quad + \frac{f_{\rho\pi\pi}^2(q^2)\sqrt{q^2}}{48\pi g^2} \left(1 - \frac{4m_{K^0}^2}{q^2}\right)^{\frac{3}{2}} \theta(q^2 > 4m_{K^0}^2).
\end{aligned} \tag{18}$$

At $q^2 = m_\rho^2$, $\Gamma_\rho = 142 MeV$. There are other channels for higher q^2 . In Ref.[6,10] Γ_ω is calculated. Put one of the two photons on mass shell, $k_2^2 = 0$, we obtain the form factor of $\pi^0 \gamma \gamma^*$

$$\begin{aligned}
F_\pi(q^2) &= f_{\pi\rho\omega}(q^2) \frac{1}{2} \left\{ \frac{-m_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2)}{q^2 - m_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2)} + \frac{-m_\omega^2 + i\sqrt{q^2}\Gamma_\omega(q^2)}{q^2 - m_\omega^2 + i\sqrt{q^2}\Gamma_\omega(q^2)} \right\}, \\
f_{\pi\rho\omega}(q^2) &= 1 + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g}\right)^2 q^2.
\end{aligned} \tag{19}$$

$f_{\pi\rho\omega}$ is the intrinsic form factor of $\pi^0 \gamma \gamma^*$. For very low momentum we obtain

$$F_\pi(q^2) = 1 + a \frac{q^2}{m_\pi^2}, \tag{20}$$

$$a = \frac{m_\pi^2}{2} \left(\frac{1}{m_\rho^2} + \frac{1}{m_\omega^2} \right) + \frac{m_\pi^2}{2f_\pi^2} g^2 \left(1 - \frac{2c}{g} \right)^2. \quad (21)$$

The first term of Eq.(21) comes from the ρ and ω poles and the second term comes from the intrinsic form factor of $\pi^0 \gamma \gamma^*$. The numerical value of a is

$$a = 0.0303 + 0.0157 = 0.046. \quad (22)$$

The first number of Eq.(22) is from the poles of vector mesons and the second number is from the intrinsic form factor(19). The contribution of the poles of vector mesons is twice of the intrinsic form factor.

The decay amplitude(4) of $\pi^0 \rightarrow \gamma \gamma$ is obtained from ABJ anomaly in the chiral limit. It is interesting to see the effect of the current quark masses, m_u and m_d , on the decay rate. The current quark mass matrix is included in Eq.(1). Taking $m_{u,d}$ into the calculation, up to the first order in current quark masses we obtain

$$\mathcal{L}_{\pi^0 \rightarrow \gamma \gamma} = -\frac{\alpha}{\pi f_\pi} f_q \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu A_\nu \partial_\lambda A_\beta, \quad (23)$$

$$f_q = 1 - \frac{1}{m} (m_u + m_d) + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 m_{\pi^0}^2, \quad (24)$$

$$\Gamma(\pi^0 \rightarrow \gamma \gamma) = \frac{\alpha^2 m_\pi^3}{16\pi^3 f_\pi^2} \left\{ 1 - \frac{1}{m} (m_u + m_d) + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 m_{\pi^0}^2 \right\}^2 \quad (25)$$

$\frac{m_u + m_d}{m}$ needs to be determined. In Ref.[11] Eq.(1) has used to predict all the 10 Gasser-Leutwyler coefficients of the ChPT[12]. The masses of the octet pseudoscalars have been derived. Up to the second order in current quark masses they are the same as the ones

obtained in ChPT. To the first order in m_q , the pion mass is expressed as[6,11]

$$m_\pi^2 = \frac{4}{f_\pi^2}(-\frac{1}{3}) < \bar{\psi}\psi > (m_u + m_d), \quad (26)$$

where $< \bar{\psi}\psi >$ is the quark condensate of three flavors. In Ref.[11] the quark condensate has been expressed as

$$-\frac{1}{3} < \bar{\psi}\psi > = 36g^4Qm^3, \quad (27)$$

and it has been determined

$$Q = 4.54.$$

It has been found in Ref.[11] that to the first order in current quark masses $f_{\pi^0} = f_{\pi^+}$. From Eqs.(26,10,11) it is obtained

$$\frac{m_u + m_d}{m} = \frac{m_\pi^2}{f_\pi^2} \frac{1}{4Q} (1 - \frac{2c}{g})^2. \quad (28)$$

The quark mass factor(24) is expressed as

$$f_q = 1 + \frac{m_\pi^2}{f_\pi^2} (1 - \frac{2c}{g})^2 \{ -\frac{1}{4Q} + \frac{g^2}{2} \} = 1 + 4.97 \times 10^{-3}. \quad (29)$$

The decay width of $\pi^0 \rightarrow \gamma\gamma$ is increased by 1%.

$$\Gamma_{\pi^0 \rightarrow \gamma\gamma} = 7.81 \times f_q^2 = 7.89 eV. \quad (30)$$

The data is $7.83(1 \pm 0.071)eV$.

To conclude, it is found that in the form factor of $\pi^0\gamma\gamma^*$ besides the vector meson poles an additional intrinsic form factor caused by quark loop is predicted. The slope of the form factor of $\pi^0 \rightarrow \gamma\gamma^*$ is predicted. The contribution of the intrinsic form factor is about 50% of the vector mesons. For the form factor of $\pi^0\gamma\gamma^*$ at high q^2 all the derivative should be taken into account. We will present the study later. The effect of current quark masses on the decay rate of $\pi^0 \rightarrow \gamma\gamma$ is investigated. It is found that the rate is increased by 1%.

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References

- [1] S.L.Adler, Phys.Rev. **177**,2426(1969); J.S.Bell and R.Jackiw, Nuovo Cimento, **60A**,47(1969).
- [2] H.J.Behrend et al., CLEO Coll., Z.Phys.,**C49**,401(1991); D.M.Asner et al., CLEO Coll., CONF95-24, EPS0188,1995.
- [3] R.M.Drees et al., Phys.Rev., **D45**,1439(1992); F.Farzanpay et al., Phy. Lett., **B278**,413(1992); H.Fonvieille et al., Phys.Lett., **B233**,651(1989); P.Gumplinger, Thesis, Oregon State Univ.(1987); J.M. Poutissou et al., Proc. of Lake Louise Winter Inst.(1987), World Scientific, Singapore; G.Tupper et al., Phys.Rev.**D28**,2905(1983);

- J.Fischer et al., Phys.Lett., **73B**,359(1978); J.Burger Thesis, Columbia Univ.(1972);
S.Devons et al., Phy.Rev.**184**,1356(1969); N.P.Samios, Phys.Rev., **121**,275(1961);
H.Kobrak, IL Nuovo Cimento, **20**, 1115(1961).
- [4] A.Gasparian et al., The PrimEX Coll., proposal.
- [5] L.G.Landsberg, Phys.Rep., **128**,302(1985); J.Bijnes,A.Bramon and F.Cornet,
Phys.Rev.Lett., **61**,1453(1988); B.Moussallam, Phys.Rev.**D51**,4939(1995);
J.Bijnes, A.Bramon, F.Cornet, Z.Phys., **C46**,599(1990);P.Maris and P.C.Tandy,
Nucl.Phys.,**A663**,401c(2000); H.R.Frank, Phys.Lett., **B359**,17(1995); H.Ito, W.Buck,
F.Gross, Phys.Lett., **B287**,23(1992); Ivanov and Lyubovitskij, hep-ph/9705423.
- [6] Bing An Li, Phys. **D52**,5165(1995); **D52**,5184(1995).
- [7] G.'t Hooft, Nucl. Phys. **B72**, 461(1974);**B75**, 461(1974); E.Witten, Nucl. Phys. **B160**,
57(1979).
- [8] J.Wess and B.Zumino, Phys.Lett., **37B**,95(1971); E.Witten, Nucl.Phys.,
B223,422(1983).
- [9] J.Gao and Bing An Li, Phys.Rev.**D61**, 113006(2000).

- [10] B.A. Li, Phys. Rev. **D55**, 1436 (1997); **D55** 1425 (1997); D.N. Gao, B.A. Li, and M.L. Yan, Phys. Rev. **D56**, 4115 (1997); B.A. Li, D.N. Gao, and M.L. Yan, Phys. Rev. **D58**, 094031 (1998);
- [11] Bing An Li, hep-ph/9810311.
- [12] J.Gasser and H.Leutwyler, Ann.Phys.(N.Y.)**58**,(1984)142; Nucl.Phys. **B250**, (1985)465; **B250**, (1985)517.

Figure Caption

Fig.1 Processes contributing to $\pi^0\gamma\gamma^*$

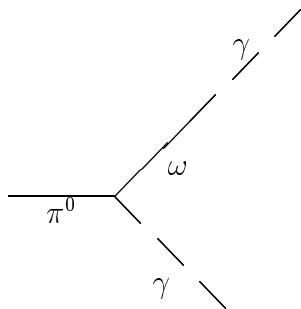


Fig.1(a)

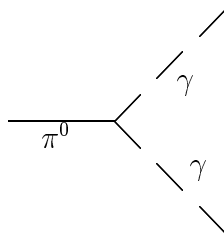


Fig.1(b)

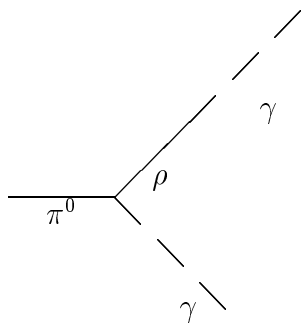


Fig.1(c)

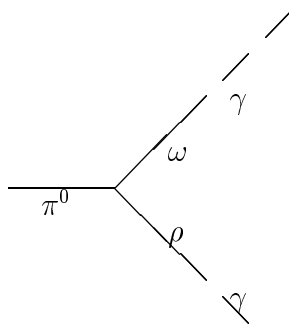


Fig.1(d)